41. **Answer (E):** Each four-digit palindrome has digit representation *abba* with $1 \le a \le 9$ and $0 \le b \le 9$. The value of the palindrome is $1001a + 110b$. Because 1001 is divisible by 7 and 110 is not, the palindrome is divisible by 7 if and only if $b = 0$ or $b = 7$. Thus the requested probability is $\frac{2}{10} = \frac{1}{5}$.

42. **Answer (D):** *Answer: 256* Rewriting each logarithm in base 2 gives

$$\frac{\frac{1}{2}\log_2 x}{\frac{1}{2}} + \log_2 x + \frac{2\log_2 x}{2} + \frac{3\log_2 x}{3} + \frac{4\log_2 x}{4} = 40.$$

Therefore $5\log_2 x = 40$, so $\log_2 x = 8$, and $x = 256$.

**OR**

For $a \ne 0$ the expression $\log_{2^a}(x^a) = y$ if and only if $2^{ay} = x^a$. Thus $2^y = x$ and $y = \log_2 x$. Therefore the given equation is equivalent to $5\log_2 x = 40$, so $\log_2 x = 8$ and $x = 256$.

43. **Answer (C):** The maximum value for $\cos x$ and $\sin x$ is 1; hence $\cos(2A - B) = 1$ and $\sin(A + B) = 1$. Therefore $2A - B = 0°$ and $A + B = 90°$, and solving gives $A = 30°$ and $B = 60°$. Hence $\triangle ABC$ is a $30-60-90°$ right triangle and $BC = 2$.

44. *Answer:* Note that $3M > (a + b) + c + (d + e) = 2010$, so $M > 670$. Because $M$ is an integer $M \ge 671$. The value of 671 is achieved if $(a, b, c, d, e) = (669, 1, 670, 1, 669)$.

45. **Answer (D):** *Answer:* 225

There are three cases to consider.

First, suppose that $i^x = (1+i)^y \ne z$. Note that $|i^x| = 1$ for all $x$, and $|(1+i)^y| \ge |1 + i| = \sqrt{2} > 1$ for $y \ge 1$. If $y = 0$, then $(1 + i)^y = 1 = i^x$ if $x$ is a multiple of 4. The ordered triples that satisfy this condition are $(4k, 0, z)$ for $0 \le k \le 4$ and $0 \le z \le 19, z \ne 1$. There are $5 \cdot 19 = 95$ such triples.

Next, suppose that $i^x = z \ne (1 + i)^y$. The only nonnegative integer value of $i^x$ is 1, which is assumed when $x = 4k$ for $0 \le k \le 4$. In this case $i^x = 1$ and $y \ne 0$. The ordered triples that satisfy this condition are $(4k, y, 1)$ for $0 \le k \le 4$ and $1 \le y \le 19$. There are $5 \cdot 19 = 95$ such triples.

Finally, suppose that $(1 + i)^y = z \ne i^x$. Note that $(1 + i)^2 = 2i$, so $(1 + i)^y$ is a positive integer only when $y$ is a multiple of 8. Because $(1 + i)^0 = 1$, $(1 + i)^8 = (2i)^4 = 16$, and $(1 + i)^{16} = 16^2 = 256$, the only possible ordered triples are $(x, 0, 1)$ with $x \ne 4k$ for $0 \le k \le 4$ and $(x, 8, 16)$ for any $x$. There are $15 + 20 = 35$ such triples.

The total number of ordered triples that satisfy the given conditions is $95 + 95 + 35 = 225$.

46. **Answer (E):** Let $N = abc+ab+a = a(bc+b+1)$. If $a$ is divisible by 3, then $N$ is divisible by 3. Note that 2010 is divisible by 3, so the probability that $a$ is divisible by 3 is $\frac{1}{3}$.

    If $a$ is not divisible by 3 then $N$ is divisible by 3 if $bc + b + 1$ is divisible by 3. Define $b_0$ and $b_1$ so that $b = 3b_0 + b_1$ is an integer and $b_1$ is equal to $0, 1,$ or $2$. Note that each possible value of $b_1$ is equally likely. Similarly define $c_0$ and $c_1$. Then

$$\begin{aligned} bc + b + 1 &= (3b_0 + b_1)(3c_0 + c_1) + 3b_0 + b_1 + 1 \\ &= 3(3b_0c_0 + c_0b_1 + c_1b_0 + b_0) + b_1c_1 + b_1 + 1. \end{aligned}$$

    Hence $bc+b+1$ is divisible by 3 if and only if $b_1 = 1$ and $c_1 = 1$, or $b_1 = 2$ and $c_1 = 0$. The probability of this occurrence is $\frac{1}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{3} = \frac{2}{9}$.

    Therefore the requested probability is $\frac{1}{3} + \frac{2}{3} \cdot \frac{2}{9} = \frac{13}{27}$.

47. *Answer:* 42

    Let $a_{ij}$ denote the entry in row $i$ and column $j$. The given conditions imply that $a_{11} = 1$, $a_{33} = 9$, and $a_{22} = 4, 5,$ or $6$. If $a_{22} = 4$, then $\{a_{12}a_{21}\} = \{2, 3\}$, and the sets $\{a_{31}, a_{32}\}$ and $\{a_{13}, a_{23}\}$ are complementary subsets of $\{5, 6, 7, 8\}$. There are $\binom{4}{2} = 6$ ways to choose $\{a_{31}, a_{32}\}$ and $\{a_{13}, a_{23}\}$, and only one way to order the entries. There are 2 ways to order $\{a_{12}, a_{21}\}$, so 12 arrays with $a_{22} = 4$ meet the given conditions. Similarly, the conditions are met by 12 arrays with $a_{22} = 6$. If $a_{22} = 5$, then $\{a_{12}, a_{13}, a_{23}\}$ and $\{a_{21}, a_{31}, a_{32}\}$ are complementary subsets of $\{2, 3, 4, 6, 7, 8\}$ subject to the conditions of $a_{12} < 5$, $a_{21} < 5, a_{32} > 5$, and $a_{23} > 5$. Thus $\{a_{12}, a_{13}, a_{23}\} \neq \{2, 3, 4\}$ or $\{6, 7, 8\}$, so its elements can be chosen in $\binom{6}{3} - 2 = 18$ ways. Both the remaining entries and the ordering of all entries are then determined, so 18 arrays with $a_{22} = 5$ meet the given conditions.

    Altogether, the conditions are met by $12 + 12 + 18 = 42$ arrays.

48. **Answer (C):** Let $A$ denote the frog's starting point, and let $P, Q$, and $B$ denote its positions after the first, second, and third jumps, respectively. Introduce a coordinate system with $P$ at $(0, 0)$, $Q$ at $(1, 0)$, $A$ at $(\cos\alpha, \sin\alpha)$, and $B$ at $(1 + \cos\beta, \sin\beta)$. It may be assumed that $0 \leq \alpha \leq \pi$ and $0 \leq \beta \leq 2\pi$. For $\alpha = 0$, the required condition is met for all values of $\beta$. For $\alpha = \pi$, the required condition is met only if $\beta = \pi$. For $0 < \alpha < \pi$, $AB = 1$ if and only if $\beta = \alpha$ or $\beta = \pi$, and the required condition is met if and only if $\alpha \leq \beta \leq \pi$. In the $\alpha\beta$-plane, the rectangle $0 \leq \alpha \leq \pi, 0 \leq \beta \leq 2\pi$ has area $2\pi^2$. The triangle $0 \leq \alpha \leq \pi, \alpha \leq \beta \leq \pi$ has area $\frac{\pi^2}{2}$, so the requested probability is $\frac{1}{4}$.

49. *Answer:* 34

    The Raiders' score was $a(1+r+r^2+r^3)$, where $a$ is a positive integer and $r > 1$. Because $ar$ is also an integer, $r = m/n$ for relatively prime positive integers $m$ and $n$ with $m > n$. Moreover $ar^3 = a \cdot \frac{m^3}{n^3}$ is an integer, so $n^3$ divides $a$. Let

$a = n^3 A$. Then the Raiders' score was $R = A(n^3 + mn^2 + m^2 n + m^3)$, and the Wildcats' score was $R - 1 = a + (a + d) + (a + 2d) + (a + 3d) = 4a + 6d$ for some positive integer $d$. Because $A \geq 1$, the condition $R \leq 100$ implies that $n \leq 2$ and $m \leq 4$. The only possibilities are $(m, n) = (4, 1), (3, 2), (3, 1), (4, 1)$, or $(2, 1)$. The corresponding values of $R$ are, respectively, $85A$, $65A$, $40A$, and $15A$. In the first two cases $A = 1$, and the corresponding values of $R - 1$ are, respectively, $64 = 32 + 6d$ and $84 = 4 + 6d$. In neither case is $d$ an integer. In the third case $40A = 40a = 4a + 6d + 1$ which is impossible in integers. In the last case $15a = 4a + 6d + 1$, from which $11a = 6d + 1$. The only solution in positive integers for which $4a + 6d \leq 100$ is $(a, d) = (5, 9)$. Thus $R = 5 + 10 + 20 + 40 = 75$, $R - 1 = 5 + 14 + 23 + 32 = 74$, and the number of points scored in the first half was $5 + 10 + 5 + 14 = 34$.

50. *Answer:* 8

The ratio between consecutive terms of the sequence is

$$r = \frac{a_2}{a_1} = \cot x,$$

so $a_4 = (\tan x)(\cot x) = 1$, and $r$ is equal to

$$\sqrt{\frac{a_4}{a_2}} = \frac{1}{\sqrt{\cot x}}.$$

Therefore $x$ satisfies the equation $\cos^3 x = \sin^2 x = 1 - \cos^2 x$, which can be written as $(\cos^2 x)(1 + \cos x) = 1$. The given conditions imply that $\cos x \neq 0$, so this equation is equivalent to

$$1 + \cos x = \frac{1}{\cos^2 x} = r^4.$$

Thus $1 + \cos x = 1 \cdot r^4 = a_4 \cdot r^4 = a_8$.